Syntax and Semantics of Programs

- **Syntax**
  - The symbols used to write a program
- **Semantics**
  - The actions that occur when a program is executed
- **Programming language implementation**
  - Syntax → Semantics
  - Transform program syntax into machine instructions that can be executed to cause the correct sequence of actions to occur

**Interpreter vs Compiler**

![Diagram of Interpreter vs Compiler](image)

**Typical Compiler**

![Diagram of Typical Compiler](image)

See summary in course text, compiler books

**Brief look at syntax**

- **Grammar**
  
  \[ 
  e \rightarrow n \mid e+e \mid e-e \\
  n \rightarrow d \mid nd \\
  d \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 
  \]

- **Expressions in language**
  
  \[ 
  e \rightarrow e-e \rightarrow e-ee \rightarrow n-nn \rightarrow nd-dd \rightarrow dd-dd \\
  \rightarrow \ldots \rightarrow 2 \ldots 3 
  \]

  Grammar defines a language
  Expressions in language derived by sequence of productions

  Many of you are familiar with this to some degree

**Theoretical Foundations**

- **Many foundational systems**
  - Computability Theory
  - Program Logics
  - Lambda Calculus
  - Denotational Semantics
  - Operational Semantics
  - Type Theory
- **Consider some of these methods**
  - Computability theory (halting problem)
  - Lambda calculus (syntax, operational semantics)
  - Operational semantics (not in book)
Lambda Calculus

• Formal system with three parts
  – Notation for function expressions
  – Proof system for equations
  – Calculation rules called reduction
• Additional topics in lambda calculus (not covered)
  – Mathematical semantics (=model theory)
  – Type systems

We will look at syntax, equations and reduction

There is more detail in the book than we will cover in class

History

• Original intention
  – Formal theory of substitution (for FOL, etc.)
• More successful for computable functions
  – Substitution --> symbolic computation
  – Church/Turing thesis
• Influenced Lisp, Haskell, other languages
  – See Boost Lambda Library for C++ function objects
• Important part of CS history and foundations

Why study this now?

• Basic syntactic notions
  – Free and bound variables
  – Functions
  – Declarations
• Calculation rule
  – Symbolic evaluation useful for discussing programs
  – Used in optimization (in-lining), macro expansion
    • Correct macro processing requires variable renaming
  – Illustrates some ideas about scope and binding
    • Lisp originally departed from standard lambda calculus
    • Haskell, JavaScript reflect traditional lambda calculus

Expressions and Functions

• Expressions
  \( x + y \)
  \( x + 2 \times y + z \)

• Functions
  \( \lambda x. (x + y) \)
  \( \lambda z. (x + 2 \times y + z) \)

• Application
  \( (\lambda x. (x + y)) \ 3 \)
  \( = 3 + y \)
  \( (\lambda z. (x + 2 \times y + z)) \ 5 \)
  \( = x + 2 \times y + 5 \)

Parsing: \( \lambda x. f \ f \ x \) = \( \lambda x. (f \ (f \ x)) \)

Higher-Order Functions

• Given function f, return function \( f \circ f \)
  \( \lambda f. \lambda x. f \ (f \ x) \)
• How does this work?
  \[
  (\lambda f. \lambda x. f \ (f \ x)) \ (\lambda y. y + 1) \\
  = \lambda x. (\lambda y. y + 1) \ ((\lambda y. y + 1) \ x) \\
  = \lambda x. (\lambda y. y + 1) \ x + 1 \\
  = \lambda x. (x + 1) + 1
  
  \]
In pure lambda calculus, same result if step 2 is altered.

Same procedure, Lisp syntax

• Given function f, return function \( f \circ f \)
  \( \text{lambda} \ (\lambda f. \ (\lambda x. (f \ (f \ x))))) \)
• How does this work?
  \[
  (((\text{lambda} \ f. \ (\lambda x. (f \ (f \ x)))) \ (\text{lambda} \ y. (+ \ y \ 1)) \\
  = (\lambda x. ((\lambda y. (+ \ y \ 1)) \ ((\lambda y. (+ \ y \ 1)) \ x))) \\
  = (\lambda x. ((\lambda y. (+ \ y \ 1)) \ ((\lambda x. (+ \ x \ 1)) \ x))))) \\
  = (\lambda x. ((\lambda y. (+ \ y \ 1)) \ ((\lambda x. (+ \ x \ 1)) \ x))))) \\
  = (\lambda x. (+ \ (\lambda x. (+ \ x \ 1)) \ x))))
  
  \]
Same procedure, JavaScript syntax

- Given function f, return function \( f \circ f \)
  
  ```javascript
  function (f) {
    return function (x) { return f(f(x)); }; }
  }
  ```

- How does this work?

  ```javascript
  function (f) { return function (x) { return f(f(x)); };; }
  function (x) { return (function (y) { return y + 1; }) (x); };
  function (x) { return ((x + 1) + 1); };
  ```

Declarations as “Syntactic Sugar”

```javascript
function f(x) {
  return x+2;
}
function (y) { return y +1; }
function (x) { return (function (y) { return y +1; }) (x); }
function (x) { return ((x +1) +1); }
```

Free and Bound Variables

- Bound variable is “placeholder”
  - Variable x is bound in \( \lambda x. (x+y) \)
  - Function \( \lambda x. (x+y) \) is same function as \( \lambda z. (z+y) \)

- Compare

  \[ \int x+y \, dx = \int z+y \, dz \quad \forall x \ P(x) = \forall z \ P(z) \]

- Name of free (unbound) variable does matter
  - Variable y is free in \( \lambda x. (x+y) \)
  - Function \( \lambda x. (x+y) \) is not same as \( \lambda x. (x+z) \)

- Occurrences
  - y is free and bound in \( \lambda x. ((\lambda y. y+2) x) + y \)

Renamed Bound Variables

- Function application
  
  ```javascript
  (\lambda f. \lambda x. f(f(x))) (\lambda y. y+2)
  ```

  - Apply twice
  - Add x to argument

- Substitute “blindly”

  \[ \lambda x. ([\lambda y. y+2] ((\lambda y. y+2) x)) = \lambda x. x+x+x \]

- Rename bound variables

  ```javascript
  (\lambda f. \lambda z. f(f z)) (\lambda y. y+x)
  = \lambda z. ([\lambda y. y+x] ((\lambda y. y+x) z)) = \lambda z. z+x+x
  ```

  Easy rule: always rename variables to be distinct

Reduction

- Basic computation rule is \( \beta \)-reduction

  \[
  (\lambda x. e_1) e_2 \rightarrow [e_2/x] e_1
  \]

  where substitution involves renaming as needed

  (next slide)

- Reduction:
  - Apply basic computation rule to any subexpression
  - Repeat

- Confluence:
  - Final result (if there is one) is uniquely determined

1066 and all that

- **1066 and All That**, Sellar & Yeatman, 1930

  1066 is a lovely parody of English history books, “Comprising all the parts you can remember including one hundred and three good things, five bad kings and two genuine dates.”

- **Battle of Hastings** Oct. 14, 1066

  Battle that ended in the defeat of Harold II of England by William, duke of Normandy, and established the Normans as the rulers of England
Main Points about Lambda Calculus

- \( \lambda \) captures “essence” of variable binding
  - Function parameters
  - Declarations
  - Bound variables can be renamed
- Succinct function expressions
- Simple symbolic evaluator via substitution
- Can be extended with
  - Types
  - Various functions
  - Stores and side-effects
  (But we didn’t cover these)

Announcements

- Homework due Wed 5PM
  - Most problems on paper
  - Upload Haskell programs using CourseWare
  - Instructions will be posted as CourseWare Announcement
- Homework grading Thurs 5:30 – 8:30 PM
  - Send email to cs242@cs.stanford.edu

Operational Semantics

- Abstract definition of program execution
  - Sequence of actions, formulated as transitions of an abstract machine
- States corresponds to
  - Expression/statement being evaluated/executed
  - Abstract description of memory and other data structures involved in computation

Structural Operational Semantics

- Systematic definition of operational semantics
  - Specify the transitions in a syntax oriented manner using the inductive nature of program syntax
- Example
  - The state transition for \( e_1 + e_2 \) is described using the transitions for \( e_1 \) and the transition for \( e_2 \)
- Plan
  - SOS of a simple subset of JavaScript
  - Summarize scope, prototype lookup in JavaScript

Simplified subset of JavaScript

- Three syntactic categories
  - Arith expressions: \( a ::= n \mid x \mid a + a \mid a * a \)
  - Bool expressions: \( b ::= a = a \mid not b \mid b \land b \)
  - Statements: \( s ::= skip \mid x ::= a \mid s ; s \mid if b then s \mathsf{else} s \mid while b do s \)
- States
  - Pair \( S = (t, \sigma) \)
  - \( t \): syntax being evaluated/executed
  - \( \sigma \): abstract description of memory, in this subset a function from variable names to values, i.e., \( \sigma : \mathsf{Var} \rightarrow \mathsf{Values} \)

Form of SOS

General form of transition rule: \[ P_1, \ldots, P_n : (t, \sigma) \rightarrow (t', \sigma') \quad (1) \]

\( P_1, \ldots, P_n \) are the \textit{conditions} that must hold for the transition to go through. Also called the \textit{premise} for the rule. These could be
- Other transitions corresponding to the sub-terms.
- Predicates that must be true.
- Calls to meta functions like:
  - \( \mathsf{get}(\sigma, x) = v \): Fetch the value of \( x \).
  - \( \mathsf{put}(\sigma, x, v) = \sigma' \): Update value of \( x \) to \( v \) and return new store.
Sample operational rules

A rule for Arithmetic Expressions

\[
\begin{align*}
(a_1, \sigma) & \rightarrow (a'_1, \sigma') [A_{\text{add}}] \\
(a_1 + a_2, \sigma) & \rightarrow (a'_1 + a'_2, \sigma') [A_{\text{add}}] \\
(a_1, \sigma) & \rightarrow (a'_1, \sigma') [A_{\text{subst}}] \\
(a_1 + a_2, \sigma) & \rightarrow (a'_1 + a'_2, \sigma') [A_{\text{subst}}]
\end{align*}
\]

How to interpret this rule?

- If the term \( a_1 \) partially evaluates to \( a'_1 \) then \( a_1 + a_2 \) partially evaluates to \( a'_1 + a_2 \).
- Once the expression \( a_1 \) reduces to a value \( n \), then start evaluating \( a_2 \).

Example:

\[
(10 + 12) + (13 + 20), \sigma \xrightarrow{A_{\text{add}}} (22 + 13 + 20), \sigma \xrightarrow{A_{\text{add}}} (22 + 33), \sigma
\]

Conditional and loops

If Then Else

\[
\begin{align*}
\text{if } (t) \text{ then } a_1 \text{ else } a_2, \sigma & \rightarrow (a_1, \sigma)[C_\text{false}] \\
\text{if } (t) \text{ then } a_1 \text{ else } a_2, \sigma & \rightarrow (a_2, \sigma)[C_\text{true}]
\end{align*}
\]

While

\[
\begin{align*}
\text{while } b \text{ do } a, \sigma & \rightarrow (a, \sigma)[C_\text{while}]
\end{align*}
\]

Sample rules

A rule for Statements

\[
\begin{align*}
(x := a, \sigma) & \rightarrow (x := a', \sigma') [C_\text{assign}]
\end{align*}
\]

How to interpret this rule?

- If the arithmetic expression \( a \) partially evaluates to \( a' \) then the statement \( x := a \) partially evaluates to \( x := a' \).
- Rule \( C_\text{assign} \) applies when \( a \) reduces to a value \( n \).
- \( \text{Put}(x, x, \sigma) \) updates the value of \( x \) to \( n \).

Example:

\[
(x := 10 + 12, \sigma) \xrightarrow{C_\text{assign}} (x := 22, \sigma) \xrightarrow{\text{Put}(x, x, \sigma)} (\sigma')
\]

Context Sensitive Rules

Similar rules

\[
\begin{align*}
(a_1, \sigma) & \rightarrow (a'_1, \sigma') [A_{\text{add}}] \\
(a_1 + a_2, \sigma) & \rightarrow (a'_1 + a'_2, \sigma') [A_{\text{add}}] \\
(a_1, \sigma) & \rightarrow (a'_1, \sigma') [A_{\text{subst}}] \\
(a_1 + a_2, \sigma) & \rightarrow (a'_1 + a'_2, \sigma') [A_{\text{subst}}]
\end{align*}
\]

- The above rules have a similar premise:
- Combine them into a single rule of the following form:

\[
\begin{align*}
2, \sigma & \rightarrow (\sigma', \sigma) \\
AC(\sigma) & \rightarrow AC(\sigma')
\end{align*}
\]

where \( AC : \bot \rightarrow a_1m + \bot = a_1n \)

Moving to full JavaScript

- Program state is represented by a triple \((H, l, t)\)
  - \(H\): program heap, mapping locations \(l\) to objects
  - \(l\): Location of the current scope object (“activation record”)
  - \(t\): expression, statement, or program being evaluated
- Note
  - All definable values (including functions) are either objects or primitive values
  - Activation records are normal JavaScript objects and variable declarations define properties of these objects
  - Instead of a stack of activation records, there is a chain of scope objects, called the scope chain

This will be a summary to show how operational semantics can be used for realistic programming language. We will not cover this semantics in detail.

Heap operations

- Each Heap object \(o\)
  - \(\{p_1 : ov_1, ... , p_n : ov_n\}\) where \(p_i\) are property names and \(ov_i\) are primitive values or heap addresses
- Operations on heap objects
  - \(\text{Dot}(H, l, p) = l_1\)
    - property \(p\) of object at location \(l\)
  - \(\text{Put}(H, l, p, l_1) = H'\)
    - Update property \(p\) of object at \(H(l)\) and return the new Heap
  - \(H', l = \text{alloc}(H, o)\)
    - Allocate object \(o\) at new location \(l\)
Scope and prototype lookup

- Every scope chain has the global object at its base
- Every prototype chain has Object.prototype at the top, which is a native object containing predefined functions such as toString, hasOwnProperty, etc

Some notation (based on ECMA Std)

• o hasProperty p
  - p is a property of object o or one of the ancestral prototypes of o
• o hasOwnProperty p
  - p is a property of object o itself
• A JavaScript reference type
  - pair written l*p where l is a heap address, also called the base type of the reference, and p is a property name

Semantics of scope lookup

ECMA 2.62:
1. Get the next object (i) in the scope chain. If there isn’t one, goto 4.
2. If i. “hasProperty” x, return a reference type l*p.x.
3. Else, goto 1
4. Return nullлист

Semantics of prototype lookup

ECMA 2.62:
1. If base type is null, throw a ReferenceError exception.
2. Else, Call the Get method, passing prop name(x) and base type l as arguments.
3. Return result(2).

Summary of Operational Semantics

- Abstract definition program execution
  - Uses some characterization of program state that reflects the power and expressiveness of language
- JavaScript operational semantics
  - Based on ECMA Standard
  - Lengthy: 70 pages of rules (ascii)
  - Precise definition of program execution, in detail
  - Can prove properties of JavaScript programs
    • Progress: Evaluation only halts with expected set of values
    • Reachability: precise definition of “garbage” for JS programs
    • Basis for proofs of security mechanisms, variable renaming, …

Imperative vs Functional Programs

- Denotational semantics
  - The meaning of an imperative program is a function from states to states.
  - We can write this as a pure functional program that operates on data structures that represent states
- Operational semantics
  - Evaluation →V and execution →E relations are functions from states to states
  - We could define these functions in Haskell

In principle, every imperative program can be written as a pure functional program (in another language)
What is a *functional* language?

- “No side effects”
- OK, we have side effects, but we also have higher-order functions ...

We will use *pure functional language* to mean
"a language with functions, but without side effects
or other imperative features."

No-side-effects language test

Within the scope of specific declarations of \( x_1, x_2, \ldots, x_n \) all
occurrences of an expression \( e \) containing only variables \( x_1, x_2, \ldots, x_n \) must have the same value.

- Example
  
  ```
  begin
  integer x=3; integer y=4;
  5*(x+y)-3
  ... // no new declaration of x or y //
  4*(x+y)+1
  end
  ```

Example languages

- Haskell
- Pure JavaScript
  ```
  function f([...], {e}, ==, {x,y,...}, first [...], rest [...], ...)
  ```
- Impure JavaScript
  ```
  x=1; ... ; x=2; ...
  ```
- Common procedural languages not functional
  - Pascal, C, Ada, C++, Java, Modula, ...

Backus’ Turing Award

- John Backus was designer of Fortran, BNF, etc.
- Turing Award in 1977
- Turing Award Lecture
  - Functional prog better than imperative programming
  - Easier to reason about functional programs
  - More efficient due to parallelism
  - Algebraic laws
    Reason about programs
    Optimizing compilers

Reasoning about programs

- To prove a program correct,
  - must consider everything a program depends on
- In functional programs,
  - dependence on any data structure is *explicit*
- Therefore,
  - easier to reason about functional programs
- Do you believe this?
  - This thesis must be tested in practice
  - Many who prove properties of programs believe this
  - Not many people really prove their code correct

Haskell Quicksort

- Very succinct program
  ```
  qsort [] = []
  qsort (xs) = qsort els_lt_x ++ [x] ++ qsort els_greq_x
  where els_lt_x = [y | y < xs, y < x]
  els_greq_x = [y | y < xs, y >= x]
  ```
- This is the whole thing
  - No assignment – just write expression for sorted list
  - No array indices, no pointers, no memory management, ...
  - Disclaimer: does not sort in place
Compare: C quicksort

```c
qsort(a, lo, hi) int a[], hi, lo;
    int h, l, p, t;
    if (lo < hi) {
        l = lo; h = hi; p = a[hi];
        do {
            while ((l < h) && (a[l] <= p)) l = l+1;
            while ((h > l) && (a[h] >= p)) h = h-1;
            if (l < h) {
                t = a[l]; a[l] = a[h]; a[h] = t;
                qsort(a, lo, l-1); qsort(a, l+1, hi);
            }
        } while (l < h);
        t = a[l]; a[l] = a[hi]; a[hi] = t;
        qsort(a, lo, l-1); qsort(a, l+1, hi);
    }
```

Interesting case study

- Naval Center programming experiment
  - Separate teams worked on separate languages
  - Surprising differences
    - Some programs were incomplete or did not run
    - Many evaluators didn’t understand, when shown the code, that the Haskell program was complete. They thought it was a high level partial specification.

Disadvantages of Functional Prog

Functional programs often less efficient. Why?

```
A     B     C     D
```

Change 3rd element of list x to y

- `(cons (car x) (cons (cadr x) (cons y (cdddr x))))`
- Build new cells for first three elements of list
- `(rplaca (cddr x) y)`
- Change contents of third cell of list directly

However, many optimizations are possible

Eliminating VN Bottleneck

- No side effects
  - Evaluate subexpressions independently
  - Example
    ```
    function f(x) { return x*y ? 1: 2; }
    g(f), f(1), f(2), ...;
    ```
- Does this work in practice? Good idea but ...
  - Too much parallelism
  - Little help in allocation of processors to processes ...
  - David Shaw promised to build the non-Von ...
- Effective, easy concurrency is a hard problem

Summary

- Parsing
  - The “real” program is the disambiguated parse tree
- Lambda Calculus
  - Notation for functions, free and bound variables
  - Calculate using substitution, rename to avoid capture
- Operational semantics
- Pure functional program
  - May be easier to reason about
  - Parallelism: easy to find, too much of a good thing
Reading

• Textbook
  – Section 4.1.1, Structure of a simple compiler
  – Section 4.2, Lambda calculus, except
    • Skip “Reduction and Fixed Points” – too much detail
  – Section 4.4, Functional and imperative languages

• Additional paper (link on web site)
  – “An Operational Semantics for JavaScript”
    • More detail than need, but provided for reference
    • Try to read up through section 2.4 for the main ideas
    • Do not worry about details beyond lecture or homework